

Magnetohydrodynamic Description of Collisionless Plasma Expansion in Upper Atmosphere

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Collisionless plasma cloud expansion into an extremely rarefied ionized medium is considered in the presence of a magnetic field when the medium is involved in the motion and the cloud is retarded due to the electromagnetic interactions. The collisionless flow is computed on the basis of a hybrid model, in which the ionic motions are considered by means of kinetic equations and the electrons are governed by a hydrodynamic equation. The numerical results of the same explosion problem according to the collisionless, hybrid, and magnetohydrodynamic models are compared and good agreement is shown. This justifies the use of hydrodynamics to describe the flows with rare collisions when formulating the general ideas on explosions in the upper ionosphere.

Nomenclature

E	= electric field
E_1	= specific internal energy
f_α	= ion distribution function species of α
H	= magnetic field
j	= current density
k	= number of freedom degrees
ℓ	= mean free path
M_A	= Alfvén Mach number
M_α	= ion mass of α species
M^-	= explosion product mass
\mathcal{M}_c	= cloud mass per unit length
n	= ion density
p	= pressure
R_g	= "hydrodynamic" retardation radius
R_H	= Larmor radius
R_m	= magnetic retardation radius
r	= radial coordinate
t	= time
u	= gas mean velocity
u_0	= initial velocity of cloud boundary
V_A	= Alfvén velocity
v	= ion velocity
γ	= adiabatic exponent
ρ	= density
φ	= azimuthal coordinate

Introduction

THE companion paper¹ presents general ideas on the blast wave propagation and the magnetic cavity formation for high-altitude explosions. The early ("hydrodynamic") stage of the explosion process is considered up to the time the shock decays strongly and the motion retards sharply. To describe the process quantitatively, several magnetohydrodynamic (MHD) models are developed: approximate quasi-two-dimensional, two-dimensional, and three-dimensional models. An explosion in the uniform atmosphere is two dimensional due to the geomagnetic field. The last model is intended for the detailed flow description in a general case for which the atmosphere is nonuniform and the geomagnetic field is inclined to the vertical. The numerical results are presented for a number

of particular cases, when the explosion occurs at not too high an altitude and the particle mean free paths at the explosion altitude are less than a flow scale R .

Nevertheless, for very high altitude explosions, the far upper boundary can extend into a region where the particle collisions are rare. Strictly speaking, the hydrodynamic equations are inapplicable under these conditions, and one should instead consider an almost free molecule flow with rare collisions (including those that lead to atmospheric gas ionization) since only charged particles are subjected to magnetic (and electric) forces.

The mathematical description of the explosion plasma expansion into the magnetized, partially ionized medium (like the upper atmosphere), and especially of flows with rare collisions, is a complicated problem, especially for three-dimensional flows. Therefore, it is very important to obtain simplified solutions giving a qualitatively correct flow field pattern. It will be shown in this paper that an MHD approach, when considering collisionless plasma expansion into the magnetized, ionized medium, can give quite reasonable qualitative and even satisfactory quantitative results. The physical reason underlying this situation is the small ion Larmor radius R_H in comparison with a characteristic flow scale R , with the ion Larmor radius playing the role of particle mean free path.

The possibility of describing collisionless plasma flow in the frame of the MHD theory will be justified in what follows when comparing solutions of the same explosion problem obtained on the basis of both the MHD theory and the kinetic equations for distribution functions of explosion product (EP) ions and ambient gas ions. Here the evaporated material of explosion device and its carrier are called, for brevity, EPs.

The results of this paper can be used to substantiate the results obtained in Ref. 1 on the basis of the MHD model for the upward flow in the extremely rarefied atmosphere.

Interaction Between Expanding Plasma and Ionized Medium in Magnetic Field

One can consider collisions (especially ionizing ones) to be absent at altitudes higher than 500–600 km where the atom density $n \leq 10^7 \text{ cm}^{-3}$ and the particle mean free path $\ell \approx 10^2\text{--}10^3 \text{ km}$. The ionizing action of the initial x-ray radiation of the explosion falls off with distance more rapidly than $1/r^2$. For weak explosions of the Argus type (explosion energy $E_i \approx 4.2 \times 10^{19} \text{ ergs} = 10^3 \text{ ton of TNT or 1 kt}$), the medium ion density $n \approx 10^5\text{--}10^6 \text{ cm}^{-3}$ can be noticeably increased by photoionization at small distances of only tens of kilometers, whereas for strong explosions of the Starfish type ($E_i = 1.4 \text{ Mt}$) these distances are hundreds of kilometers. In an case, only charged particles that are in the atmosphere before the EP cloud expansion begins can be involved in the motion.

For the collisionless case, the medium ion acceleration is provided by the geomagnetic-field-generated electric fields. A so-called

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magnetolaminar mechanism takes place²; i.e., as a result of displacement of the magnetic field by the plasma cloud, an electric field normal to the plasma expansion velocity and magnetic field directions is induced. The electric field accelerates the medium ions, and the Lorentz force makes them move in the main direction of the cloud expansion. As a result of our computations another mechanism is also shown to dominate under some conditions. Since the Larmor radii of electrons and ions are different, a charge separation occurs and the polarization electric field arises near the boundary of the EP cloud and in a region attached to the front of the collisionless shock wave that propagates in the medium. The field accelerates the medium ions just along the EP cloud expansion direction.

In principle, one can imagine a mechanism to generate the turbulence and stochastic fields, but such a mechanism is not taken into account in our computations. This is a quite special and complicated problem and is not considered here. As to so-called abnormal ionization due to dissipation of plasma oscillations,³ one can deal only with the collisionless electron gas heating up to energies sufficient for ionization. The ionization occurs always in a "normal" way, the ionization cross section being at least an order of magnitude less than the gas kinetic cross section. At the altitudes under consideration, the ionization frequency is $\nu_i \sim 10^{-1} \text{ s}^{-1}$, so that such ionizing collisions are extremely rare. However, there is a region at lower altitudes where the abnormal ionization may be of importance.⁴

Hybrid Model

The hybrid model is used to study the collisionless plasma motion in a magnetic field. This model describes ions by means of the velocity distribution function $f(\mathbf{r}, \mathbf{v}, t)$, the electrons being considered hydrodynamically.^{5,6} The typical scale on which the macroscopic parameters (the plasma density, magnetic field) vary appears to be the ion Larmor radius $R_H \sim M_e c u / e H$. The electron Larmor radius R_{He} is much less than R_H due to the small electron mass, $m \ll M_e$. Of course, the electron distribution function changes sharply on a small scale R_{He} , but we do not consider such details. If we are interested in the behavior of those macroscopic parameters that vary on a scale R_H , it is sufficient to consider only the motion of the electron small-scale rotation centers. In this case one can consider the electrons as a gas and describe their motions hydrodynamically.

All of the estimates show that the plasma under the conditions in question is essentially quasineutral everywhere, although there are narrow space charge regions. These regions are located near the EP boundary as well as near the collisionless shock in the background medium. The scales of these regions are essentially less than R_H , and therefore, they are not considered here. (In the case of EP expansion into vacuum the problem of the space charge sheath was discussed in Ref. 7). Estimating the ionization degree of the initially strongly ionized EP cloud, one can show that the cloud consists mainly of the singly ionized ions. To simplify, let us consider all EP ions to be singly ionized (as well as the ionospheric ions). Then the densities of ions and electrons in quasineutral plasma are equal.

Let the EP ions have the index $\alpha = 1$, whereas the index of the background ions is $\alpha = 2$. The ion distribution functions f_α are governed by the Vlasov kinetic equation

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial f_\alpha}{\partial \mathbf{r}} + \frac{e}{M_\alpha} \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{H}}{c} \right) \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = 0 \quad (1)$$

The electric and magnetic fields satisfy the Maxwell equations

$$\text{rot } \mathbf{H} = \frac{4\pi}{c} \mathbf{j}, \quad \text{rot } \mathbf{E} = -c^{-1} \frac{\partial \mathbf{H}}{\partial t} \quad (2)$$

in which the displacement current is omitted.

The current density in the singly ionized quasineutral plasma is

$$\mathbf{j} = en(\mathbf{u} - \mathbf{u}_e) \quad (3)$$

where the density and the ion mean velocity are defined as

$$\begin{aligned} n &= \sum n_\alpha, & n\mathbf{u} &= \sum n_\alpha \mathbf{u}_\alpha \\ n_\alpha &= \int f_\alpha d\mathbf{v}, & n_\alpha \mathbf{u}_\alpha &= \int \mathbf{v} f_\alpha d\mathbf{v} \end{aligned} \quad (4)$$

The mean electron velocity \mathbf{u}_e satisfies the equation

$$\mathbf{E} + [\mathbf{u}_e \times \mathbf{H}] / c = 0 \quad (5)$$

One can interpret equality (5) as follows. The electromagnetic force acting on the electrons and ions does not depend on the particle mass; therefore light electrons are accelerated much more than ions. This would lead to charge separation and would violate the quasineutrality condition if the strong Coulomb interaction would not prevent this from occurring; i.e., an appropriate electric field arises to pressure quasineutrality. The mean force itself, which so strongly accelerates the electron gas, has to disappear or, more exactly, its electric component has to compensate the Lorentz force within the accuracy of the order of m/M . This fact is expressed by the approximate equality (5). System (1–5) is a closed system used to investigate the collisionless plasma motion in an external magnetic field.

Problem Formulation and Aims of Analysis

The computations according to the hybrid model are time consuming. Using the computers available, it is difficult to compute a two-dimensional plasma cloud expansion up until times of interest. In Refs. 8 and 9 the computations are performed up to a time moment when a perturbation goes away through the background plasma at a distance of a few R_H . (It is shown by these computations that the magnetic field line pattern is similar to that obtained with the MHD model; Fig. 2 in Ref. 1.) It is just the Larmor radius that determines the length scale at which the macroscopic parameters ($n, \mathbf{u}, \mathbf{H}$) change significantly both on the EP background interface and in the wave propagating through the background plasma. Hence, when computing, the spatial step size has to be taken essentially less than R_H . Given the conditions of the problem, R_H is of order of a few kilometers. This means that to obtain sufficiently complete results on the process in question it is desirable to carry out the computations up to the stage when the wave covers a distance $R \simeq 10^2 R_H$. Since we have no possibility to do this for two-dimensional explosion, let us consider instead the one-dimensional problem (with cylindrical symmetry) to clarify the main physical peculiarities of the collisionless process. Our aim is to compare the results with those of the MHD model.

Thus, let the initially uniform magnetic field \mathbf{H}_0 permeate the uniform "cold" background plasma of the density $n_0 = \rho_0 / M_2$ and the plasma cloud of the ion mass M_1 be symmetrically expanded from the axis parallel to \mathbf{H}_0 . The cloud is of radius R_1 , its initial density ρ_1 is constant, and its expansion velocity due to the inertial expansion is linearly distributed along the radius, $u_1 = u_0(r/R_1)$.¹⁰ The cloud mass $\mathcal{M}_c = \rho_1 \pi R_1^2$, and its kinetic energy $E_1 = \mathcal{M}_c u_0^2 / 4$ referred to the cylinder height unit size. The values of ρ_1 and R_1 can be arbitrarily chosen, but the mass \mathcal{M}_c and the velocity u_0 have to be specified in such a way as to correspond to the real "spherical" process of interest. It is clear that, after the cloud is expanded at a sufficiently large distance, the solution "forgets" ρ_1 and R_1 and will "remember" only their combination \mathcal{M}_c . There are three typical lengths in the above-formulated problem: 1) the Larmor radius of the background ions, e.g., R_{H2} ; 2) the hydrodynamic retardation radius $R_g = (\mathcal{M}_c / \pi \rho_0)^{1/2}$, (Ref. 1); and 3) the magnetic retardation radius $R_m = (8E_1 / H_0^2)^{1/2}$.

When making the equation and initial conditions dimensionless (by normalizing all the quantities with the natural scales u_0, H_0, ρ_0 , and one length, say R_{H2}), one can see that the solutions are self-similar if we keep the length ratio $R_{H2} : R_g : R_m$ (and, of course, M_1 / M_2) fixed. In this case, there are two dimensionless similitude parameters, the Alfvén Mach number u_0 / V_A ($V_A = \sqrt{H_0^2 / 4\pi \rho_0}$), which in the cylindrical case is $M_A = u_0 / V_A = 2^{1/2} (R_m / R_g)$, and $\delta = R_g / R_{H2}$. It is desirable to retain these parameters when proceeding from the "spherical" problem to the cylindrical one. The cylindrical version would look even more like the spherical one if we use the scaling quantities ρ_0, u_0 and the smallest of the retardation radii typical for the spherical problem. Since we are interested in collisionless expansion involving the medium in the motion, we consider the cases for which $R_g < R_m$.

Computational Results

The equations are solved by the particle-in-cell method. Instead of the kinetic equation integration, the trajectories of a large number of the macroscopic particles (each of these particles in turn consist of the large number of ions) are computed. The particle trajectory is described by the equations

$$\frac{dr}{dt} = v_r$$

$$\frac{dv_r}{dt} = \frac{v_\varphi^2}{r} + \frac{e}{M_\alpha} \left(E_r + \frac{v_\varphi H}{c} \right)$$

$$\frac{dv_\varphi}{dt} = -\frac{v_\varphi v_r}{r} + \frac{e}{M_\alpha} \left(E_\varphi - \frac{v_r H}{c} \right)$$

the centripetal acceleration being taken into account. At each time step the new position and velocity of the macroparticle are computed according to the above equations, with the results of the previous step taken as the initial conditions. Then we average the results over the set of the macroparticles so that the macroscopic velocities of both plasma species that determine the current density are found. The new values of the field are found from the Maxwell equations. The details of the complicated computational process, computational schemes, the averaging procedure, etc., are described in Ref. 11.

As an example, we consider the cylindrical case corresponding to the spherical explosion with the kinetic energy $E = 100$ kt and the mass of EP, $M = 1$ ton, in the medium of atomic oxygen ions for which we have $\rho_0 = 2.53 \times 10^{-16}$ g/cm³, $n_0 = 0.96 \times 10^7$ cm⁻³. (The cloud energy E is the difference between the total explosion energy E_t and the energy radiated by the x-rays at the initial moment.) When the uniform sphere of EP is expanded, then for the linear initial expansion velocity distribution along the radius the velocity of the sphere boundary is $u_0 = \sqrt{10E/3M} = 1260$ km/s. When computing the cylindrical counterpart of the spherical process, we take the same values of ρ_0 , n_0 , u_0 , and ion mass ratio $M_1/M_2 = 1.7$. To find the mass per unit length for the equivalent cylindrical cloud of EP, we set the hydrodynamic retardation radius to be the same as in the spherical case, $R_g = 97$ km. This gives the mass $\mathcal{M}_c = \rho_0 \pi R_g^2 = 7.55 \times 10^{-2}$ g/cm and the EP initial energy $E_1 = \mathcal{M}_c u_0^2/4 = 3 \times 10^{14}$ ergs/cm. For the parameters chosen, the Alfvén velocity and the Mach number are $V_A = 80$ km/s and $M_A = u_0/V_A = 14.3$, respectively, and the ion Larmor radius is $R_{H2} = 4$ km. The initial radius of the EP cylindrical cloud is taken to be $R_1 = 2R_{H2} = 8$ km, the EP initial density being $\rho_1 = 590\rho_0$.

The results are shown in Figs. 1–3, where the radial distribution of various quantities are given for time $t = 0.16$ s. At this time, the collisionless shock wave in the medium extends to a distance $R = 37R_{H2} = 150$ km from the axis, whereas the EP boundary covers a distance $21R_{H2} = 84$ km. Due to the retardation of the forward portion of the cloud front, a part of its mass (of order of a half of the mass at this moment) forms a shell near the front boundary. In a region between the shock and the EP boundary the background gas is compressed, the magnetic field being displaced by the EP cloud; however, since the field is frozen, it grows in the background gas. An induced azimuthal electric field is generated in a region of increased magnetic field, and everywhere it is pointed in the same direction. The radial electric field is also nonvanishing in this region only and, being irregular, is directed mainly forward. It is always directed forward at the disturbance forefront, accelerating the background ions in the plasma expansion direction.

The velocities, densities, and electric field distributions are strongly irregular, and the spatial scale of this irregularity is of order of the Larmor radius or an even smaller value. This accounts for the peculiarities of the computational procedure based on detailed consideration of the individual large particle motions. The oscillatory structure depends on the choice of the particles and on the computational step along the radius, but the front velocities and the mean values of the macroscopic parameters do not depend essentially on the above-listed points. This allows us to be sure that the averaged characteristics are correct.

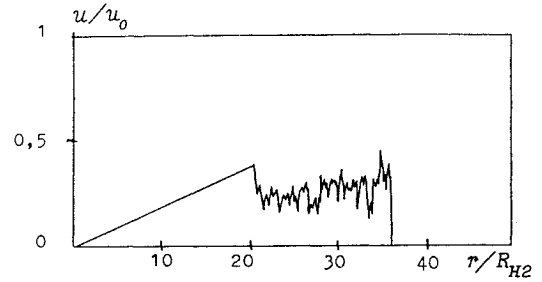


Fig. 1 Collisionless cylindrical-symmetric ion cloud expansion into rarefied ionized medium in magnetic field $H = 0.5$ Oe according to hybrid model. Medium ion density $n_0 = 10^7$ cm⁻³, $\rho_0 = 2.53 \times 10^{-16}$ g/cm³, and ratio of cloud ion mass to that of medium is 1.7. Cloud mass is referred to cylinder mass per unit length ($\mathcal{M}_c = 7.55 \times 10^{-2}$ g/cm), and boundary initial velocity ($u_0 = 1260$ km/s) corresponds approximately to case of explosion with kinetic energy $E_k = 100$ kt and EP mass of 1 ton in same medium. Radial velocity distribution along radius is given at time $t = 0.16$ s. Velocity is normalized with respect to u_0 , and radial distance is normalized by ion Larmor radius in medium, $R_{H2} = 4$ km.

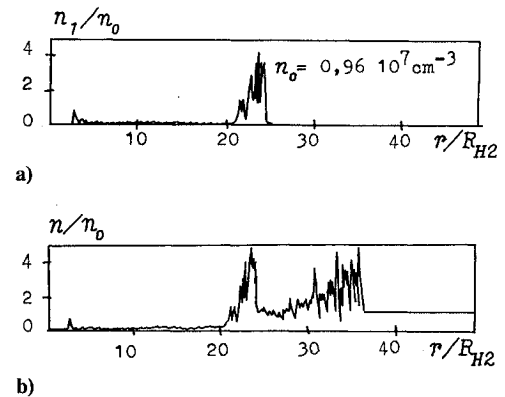


Fig. 2 a) Distribution of EP ion density and b) total EP and medium ion density, with densities being normalized by undisturbed density of medium, n_0 .

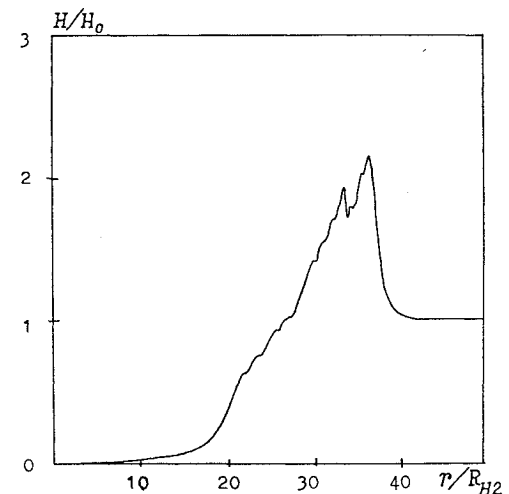


Fig. 3 Magnetic field distribution normalized by undisturbed magnetic field H_0 .

At the initial stage of the process the fore EP ions pass ahead of the background ions; i.e., we have a situation in which the EP front propagates through a quiescent background medium, and different gases are mixed. At this stage the background ions are accelerated mainly by the azimuthal electric field. Then, turning under the Lorentz force action, they acquire the radial velocity. At the later stage when the disturbance front in the medium propagates ahead of the EP boundary and the strong radial electric field arises, the motionless background ions acquire a velocity due to both azimuthal and radial

fields, the action of the radial field being dominant. In contrast to the azimuthal field, the radial electric field is a space-charge field. This is seen from Eq. (5) since electromagnetic induction Eq. (2) provides the azimuthal field induction rather than the radial one.

Comparison of Results Obtained According to Hybrid and Magnetohydrodynamic Models

In Fig. 4 the computational results for the same conditions assumed in Figs. 1–3 are given, but they are obtained by using the MHD model equations with $\gamma = \frac{5}{3}$. Since there is no concept of a Larmor radius in the MHD model, the scale is defined according to R_{H2} in the hybrid model. As one can see comparing Figs. 1–4, the agreement between the distributions is excellent with regard to both the positions (and velocities) of the fronts and the magnitudes of u and H . The H distribution characterizes the gas density distribution due to the frozen field as well. As will be seen, one can expect even better agreement with the results of the hybrid model for $\gamma = 2$, but the MHD solutions themselves differ so insignificantly for $\gamma = \frac{5}{3}$ and $\gamma = 2$ that further corrections are hardly worthwhile.

This agreement is not surprising if we remember how the equations for a perfect fluid are derived from the kinetic equation. The kinetic Boltzmann equation [the left-hand side of which coincides with Eq. (1) whereas the right-hand side involves the collision integral] is multiplied by M_α , $M_\alpha v_i$ and $M_\alpha v^2/2$ and is integrated with respect to v . The integrals on the right-hand side vanish identically (if we take into account the elastic collisions only), whereas the integration of the left-hand side results in the continuous medium equations that are the conservation laws for mass, momentum, and energy. The constitute the closed system of gasdynamic equations for ρ , u , p , ε and the state equation $\varepsilon(p, \rho)$, provided we postulate isotropy for the chaotic motion of the particles $\langle v_i v_k \rangle = \langle v_i^2 \rangle \delta_{ik}$, $\langle v_i^2 \rangle = \langle v^2 \rangle / 3$, so that the densities of the momentum and energy fluxes can be expressed in terms of ρ , u , p , and ε . This postulate

enables one to avoid manipulations with the collisional quantities. It is only necessary to consider the ultimate result of collisions, provided the particle mean free path ℓ is essentially less than a typical flow scale R .

If there are no collisions, but there is an external magnetic field, the ion path turning is a mechanism of the chaotization. The Larmor radius R_H plays the role of a mean free path. At a late stage of the process, when $R \gg R_H$, there are ions with various rotational velocity directions at each point because they begin to rotate at different points and at different times. Again, this makes it possible to postulate the isotropy of the chaotic velocities, and after averaging, the equations of the collisionless hybrid model are transformed into the MHD equations. Since in the hybrid model the difference between the macroscopic velocities of the ions and electrons is taken into account as well as the finite plasma density n [the equality (3)], H is determined by the equation

$$\frac{\partial H}{\partial t} = \text{rot}[u \times H] - \frac{c}{4\pi e} \frac{\text{rot}[\text{rot} H \times H]}{n} \quad (6)$$

which follows from Eqs. (2), (3), and (5). For the parameters of interest the second term on the right side of Eq. (6) is small. One can omit this term and use the standard equation of MHD theory.

In the case considered here of a cylindrical cloud, the particles have no axial velocities; therefore, the chaotic motion has $k = 2$ translation degrees of freedom only instead of the usual $k = 3$. This is why the adiabatic exponent is $\gamma = [(k/2) + 1]/(k/2) = 2$, instead of $\gamma = \frac{5}{3}$, as was noted above. This can be considered as a modification of the ChGL theory²¹ as applied to a two-dimensional process in which there is no pressure in the magnetic field direction. Since the solutions of the MHD equations with $\gamma = \frac{5}{3}$ and $\gamma = 2$ differ only slightly, the approximate extension of the standard MHD theory with the isotropic pressure and $\gamma = \frac{5}{3}$ to the complicated three-dimensional flows with curved magnetic fields may be justified.

Remarks on Case of Rare Collisions

We have established a basis for considering the collisionless gasdynamic expansion of a cloud into an ionized medium with an external magnetic field by use of an MHD model, which is simpler than a hybrid (kinetic) model. In so doing, one should ignore the neutral background component, which does not take part in the process. In the case of rare collisions the motion itself could be considered according to an MHD model; however, a question arises regarding how many atoms are involved in the process. Strictly speaking, to solve this problem (and only this problem), one has to proceed from the MHD description to that of the kinetic one. But compromises are possible when we use a simpler gasdynamic approach (instead of the complicated kinetic plasma description). This approach consist of introducing a source of the background mass involved in the motion into the gasdynamic equations, with the background mass providing retarding action on the moving gas. Here one should take into account the effects of the gasdynamic collisions, charge transfer, electron heating (both the usual one due to the heat exchange when colliding and the abnormal one due to dissipation of the plasma oscillations), and atomic ionization by the electron impact, since all of these effects are of comparable rates.

Conclusion

The collisionless explosion plasma expansion into an ionized magnetized medium is computed according to the kinetic equation. Comparison with the same problem solved in the frame of the MHD equations shows good agreement. The physical reason that the gasdynamics equations, which are formally inapplicable to collisionless flow, give acceptable results is that turning of ions around the magnetic field lines plays a role of particle collisions. When the ion free path and the ion Larmor radius are small, both effects (i.e., collisions and turning) lead to chaos, and this is what is necessary for the gasdynamics equations to be valid. The results obtained justify the application of the MHD model to compute explosions in the upper atmosphere. The MHD models were developed in the companion paper.¹ These models are essentially simpler than the ones based on a kinetic description of particle motion. The problem

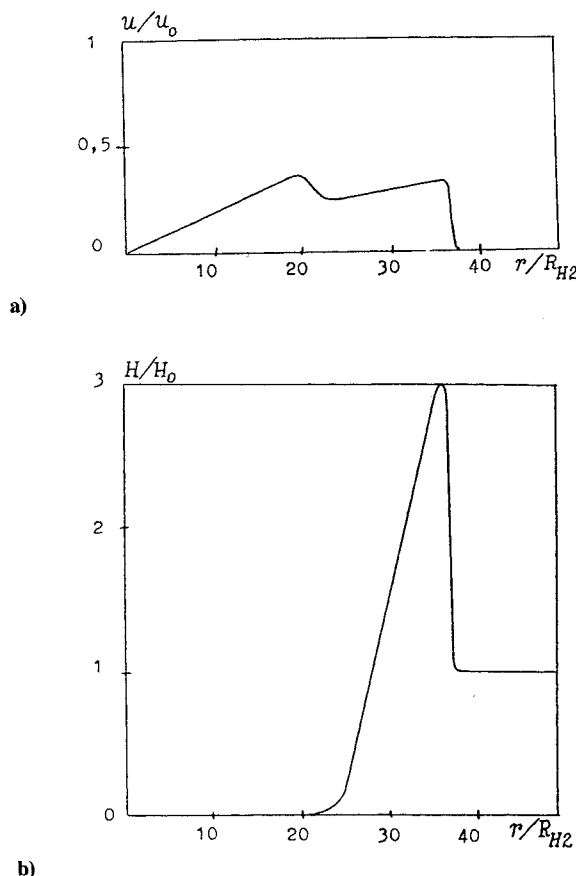


Fig. 4 Same problem as in Figs. 1–3 computed by MHD equations with $\gamma = \frac{5}{3}$: distribution of a) velocity and b) magnetic field normalized by u_0, H_0 at same time moment as in Figs. 1 and 3. To make comparison with Figs. 1 and 3 more convenient, distance r is divided by $R_{H2} = 4$ km.

concerned with ionization of the atmosphere requires an additional analysis since only charged particles involved in the motion caused by electromagnetic interactions can "participate" in the collisionless flow in the geomagnetic field. The problem of abnormal ionization is of special importance.

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